

## **APPENDIX M**

1. Input the preprocessed design matrix  $X$  of dimension  $M \times K$
2. Input the observed outcome vector  $Y$  of dimension  $M \times 1$
3. Compute the SVD of  $X$ , *i.e.*,  $X = USV^T$ , where  $U = (U_1, U_2, \dots, U_k)$ ,  $V = (V_1, V_2, \dots, V_k)$  are left and right singular vectors, respectively
4. Compute the solution vector of the model as

$$\beta = \sum_{i=1}^K \left( \frac{U_i Y}{\sigma_i} \right) V_i$$

where  $\sigma_i$  are the singular values, and  $U_i Y$  are the vector dot product between  $U_i$  and  $Y$ . In one embodiment, in order to avoid some potential overflow that may occur in this step due to possible small singular values, a threshold (e.g.,  $10e-5 * \max(\text{singular values})$ ) to eliminate small values is implemented.

A corresponding prototype code is listed below:

```
load X.dat;
load y.dat;
y = y';
[m, n] = size(X);
[U,S,V] = svd(X,0);
sigma = 10E-5 * S(1,1);
k = 0;
for i = 1:n
    if (S(i,i) >= sigma)
        k = k + 1;
    end
end
beta = 0;
for i = 1:k,
    beta = beta + ((U(:,i)'*y)/S(i,i))*V(:,i);
end
beta
```